## Composite Hugoniot Synthesis Using the Theory of Mixtures

density,  $\tilde{\rho}_{\alpha}$ . No relation such as Equation (37) can be obtained for the constituent compression in the physical configuration,  $\tilde{\rho}_{\alpha}^{-}/\tilde{\rho}_{\alpha_{0}}$ . When  $\hat{c}_{\alpha}$  is not  $\sigma^{\alpha}$ 

zero, Equation (37) must be replaced by

$$\eta_{\alpha} = \eta \left( 1 + \frac{\hat{c}_{\alpha}}{\rho_{\alpha_0} U} \right)$$
(38)

We can now investigate the question of a phase transformation in one of the constituents. Suppose constituent  $S_1$  transforms into constituent  $S_2$  at shock pressures in excess of  $\tilde{P}^*$ . We assume that no  $S_2$  is initially present leading the shock. That is

$$\rho_{2_0} = 0$$
(39)

We assume that, following the shock, all of constituent  $S_1$  has been transformed to  $S_2$  so that

$$\rho_1^- = 0 \tag{40}$$

Using Equation (40) in Equation (25), we obtain

$$\hat{c}_1 = -\rho_{1_0} U, \quad \text{for} \quad \widetilde{P}_{\alpha} \ge \widetilde{P}^*$$
(41)

while use of Equation (39) in Equation (25) gives

$$\hat{c}_2 = \rho_2(U - v^-), \quad \text{for} \quad \widetilde{P}_{\alpha} \ge \widetilde{P}^*$$
(42)

Then, if no other mass exchange occurs, Equation (13) implies that

$$\rho_2(U - v^-) = \rho_{10}U \tag{43}$$

whenever  $\widetilde{P}_{\alpha}^{-}$  exceeds the transformation pressure,  $\widetilde{P}^{*}$ . In this case, we have

$$\frac{\rho_2}{\rho_{1_0}} = \eta_3 = \eta_4 = \dots = \eta_k = \eta$$
(44)

More complex interactions, including chemical reactions among constituents within the shock surface, can be treated in a similar manner. It is necessary only to assume constitutive relations for the supplies,  $\hat{c}_{\alpha}$ , obeying Equation (12)

(13).

Next, we turn to the physical configuration and assume that the constituent crystal pressures,  $\widetilde{P}_{\alpha}$ , are all equal, provided  $S_{\alpha}$  exists behind the shock. That is

 $\widetilde{P}_{\alpha}^{-} = \widetilde{P}^{-}, \quad \text{for all } \alpha \text{ such that } \rho_{\alpha}^{-} \neq 0$  (45)

Then Equations (23) and (30), together with this assumption, imply that

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$$P^- = \widetilde{P}^- \tag{46}$$

That is, the total mixture pressure must equal the crystal pressure in each constituent. The momentum jump equation [Equation (26)] can then be written as

$$n_{\alpha}^{-}\widetilde{P}^{-} + m_{\alpha} = \rho_{\alpha}^{-}(U - v^{-})v^{-}$$
(47)

Summing this equation over  $\alpha$  and using Equations (14) and (16), we obtain the usual momentum jump equation for the whole mixture.

$$\widetilde{P}^{-} = \rho^{-} (U - v^{-}) v^{-}$$
(48)

Our assumptions of equal particle velocities and equal crystal pressures in all extant constituents have implied the form of the momentum supply functions,  $\hat{m}_{\alpha}$ . Eliminating the quantity  $(U - v^{-})v^{-}$  between Equations (47) and (48) vields

$$\hat{m}_{\alpha} = \widetilde{P}^{-}(c_{\alpha}^{-} - n_{\alpha}^{-}) \tag{49}$$

where  $c_{\alpha}^{-}$  is the concentration  $\rho_{\alpha}^{-}/\rho^{-}$  following the shock. Using Equation (38), we can write  $c_{\alpha}^{-}$  as

$$c_{\alpha}^{-} = c_{\alpha_{0}} \left( 1 + \frac{\hat{c}_{\alpha}}{\rho_{\alpha_{0}} U} \right)$$
(50)

Note that, if no mass transfer occurs, the concentration will not change crossing the shock.

The energy balance relation, Equation (27), may also be simplified by these assumptions. If  $S_{\alpha}$  does not vanish behind the shock, we can divide Equation (27) by the volume fraction,  $n_{\alpha}^{-}$ , to obtain

$$\widetilde{P}^{-}v^{-} + \frac{\widetilde{\epsilon}_{\alpha}}{n_{\alpha}^{-}} = \widetilde{\rho}_{\alpha}^{-}(U - v^{-})\left(e_{\alpha}^{-} + \frac{1}{2}(v^{-})^{2}\right) + \widetilde{h}_{\alpha}^{-}$$
(51)

where Equations (17), (19), (23), (28), and (45) have all been used. Also, summing Equation (27) over  $\alpha$  and taking Equations (15), (30), (31), and (32) into account, we obtain the usual energy jump equation for the whole mixture.

$$\widetilde{P}^{-}v^{-} = \rho^{-}(U - v^{-})\left(\epsilon^{-} + \frac{1}{2}(v^{-})^{2}\right) + h^{-}$$
(52)

We point out here that the constituent heat flux,  $h_{\alpha}^{-}$ , does not account in any way for transfer of heat between constituents. The action of  $h_{\alpha}^{-}$  is entirely restricted to  $S_{\alpha}$ . Heat may be transferred among the constituents, however,